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The domain-wall/QFT correspondence

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Abstract: We extend the correspondence between adS-supergravities and superconformal field theories on the adS boundary to a correspondence between gauged supergravities (typically with non-compact gauge groups) and quantum field theories on domain walls.

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Contents

1. Introduction

Evidence is currently accumulating for a conjectured equivalence between M-theory or IIB superstring theory in an anti-de Sitter (adS) background and a superconformal field theory (SCFT) at the adS boundary [1] (for related earlier work see [2], [3]). The isometry group of the KK vacuum acts as the superconformal group on the SCFT at the adS boundary in the manner envisaged in earlier studies of singleton field theories [4] and branes 'at the end of the universe' [5]. In the new approach the SCFT describes the dynamics of N near-coincident branes in the low-energy limit (or equivalently in the limit of decoupling gravity). This limit corresponds to the near-horizon limit of the corresponding brane solution of D=11 or IIB supergravity, which turns out to be one of the well-known Kaluza-Klein (KK) compactifications to an adS spacetime [6]. The (conjectured) equivalence of the bulk supergravity theory (more precisely the underlying M-theory or superstring theory) to the SCFT boundary theory nicely illustrates the holography principle[1] that is widely believed to be a feature of any consistent theory of quantum gravity [7]. However, this principle also suggests that the adS/CFT correspondence is just a special case of a more general correspondence between supergravity theories (at least those that are effective theories for some consistent quantum theory) and quantum field theories in one lower dimension.

[1]The equivalence between a KK supergravity and a SCFT at the adS boundary was actually already conjectured in [5] but it was then viewed not as an illustration of holography but as a generalization to branes of the fact that the conformal field theory on a string worldsheet encodes D=10 supergravity scattering amplitudes.

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One clue to a possible generalization of the adS/CFT correspondence is the fact that the adS metric in horospherical coordinates is a special case of a domain wall metric [8]. The isometry group of the generic D-dimensional domain wall spacetime is the Poincaré group in $(D-1)$ dimensions. In the supergravity context a domain wall typically preserves half the supersymmetry and hence admits a super-Poincaré isometry group. If the supergravity theory arises via KK compactification on some space B then the KK domain wall 'vacuum' is additionally invariant under the isometries of B. The adS case is special in that the KK vacuum is invariant under a larger adS_D supergroup that contains as a proper subgroup the product of the $(D-1)$-dimensional Poincaré group with the isometry group of B. This precisely corresponds to the fact that a $(D-1)$-dimensional superconformal field theory is a special case of a $(D-1)$-dimensional supersymmetric quantum field theory (QFT), for which the invariance group is generically just the product of the $(D-1)$-dimensional super-Poincaré group with its R-symmetry group. The latter is naturally identified with the isometry group of B which is also, according to Kaluza-Klein theory, the gauge group of the D-dimensional supergravity theory admitting the domain wall solution.

We are thus led to investigate whether there are KK compactifications of D=10 or D=11 supergravity theories to domain wall 'vacua' analogous to compactifications to an adS spacetime. One cannot expect such a solution to preserve all supersymmetries but experience with branes suggests that it might preserve 1/2 supersymmetry. In fact, a number of examples of this type are already known and, in close analogy to the adS case, they can be interpreted as the 'near-horizon' limits of brane solutions of D=11 or D=10 supergravity theories. For example, the 'near-horizon' limit of the NS-5-brane of N=1 supergravity yields an S^3 compactification of N=1 D=10 supergravity to a D=7 spacetime that can be interpreted as a domain-wall solution of the effective D=7 theory [6, 9, 10]. The latter can be identified with the $SU(2)$-gauged D=7 supergravity [11] coupled to an $SU(2)$ super-Yang-Mills (SYM) multiplet, in accord with the fact that the isometry group of S^3 is $SU(2) \times SU(2)$ (the same S^3 compactification is also applicable to the NS-5-branes of Type II supergravities but we postpone discussion of these cases). A new example that we shall discuss here is an S^2 compactification of IIA supergravity to a D=8 domain wall spacetime. It can be found as a 'near-horizon' limit of the IIA D6-brane solution. The effective D=8 supergravity theory is the $SU(2)$-gauged maximal supergravity.[2] As adS supergroups exist only for $D \leq 7$ it follows that this, or any other, D=8 supergravity theory cannot admit a supersymmetric adS vacuum (by which we mean one for which the isometry group is an adS supergroup). However, as we shall show, it does admit a 1/2 supersymmetric domain wall vacuum, and this domain wall solution is precisely the one found in the 'near-horizon' limit of the D6-brane.

[2] This was originally obtained as an S^3 reduction of D=11 supergravity [12], but it can be viewed as an S^2 compactification of IIA supergravity in which the RR 2-form field strength is proportional to the volume of the 2-sphere.

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The term 'near-horizon' is placed in quotes in the above paragraph because what is essential for the argument is not the existence of a horizon but rather of a second asymptotic region near the core of the brane into which spacelike geodesics can be continued indefinitely. This is a feature of the NS-5-brane in string-frame because in this frame the singularity at the core is pushed out to infinity. It is not a feature of the D6-brane in either the string frame or the Einstein frame but there *is* a frame, the 'dual' frame, in which the singularity is again pushed out to infinity, and in this frame one finds a 'vacuum' solution of IIA supergravity with an $adS_8 \times S^2$ 10-metric [10, 13]. The 'dual' frame can be defined for general p as the one for which the tension of the dual $(6-p)$-brane is independent of the dilaton. This implies that in the effective action the function of the dilaton multiplying the Einstein term is the same as the function multiplying the dual of the $(p+2)$-form field strength. It is a general property of p-brane solutions, except when $p=5$, that the dual frame metric is the product of an adS space with a sphere [14, 10, 13]. When $p=5$ one finds that the adS space is replaced by a Minkowski spacetime, so this case requires a separate discussion. In all cases, however, there is an 'internal' infinity in the dual metric, and hence a second asymptotic region near the p-brane core. The importance of this is that the effective supergravity in this asymptotic region must, to the extent to which the supergravity approximation remains valid, describe the 'internal' dynamics on the brane.[3]

One thus arrives at the (tentative) conclusion that the QFT describing the 'internal' dynamics of N coincident branes is equivalent to the supergravity theory in the 'near-horizon' region of the corresponding supergravity brane solution (but in a given region of the parameter space there is only one weakly coupled theory). This is essentially the argument of [1] in support of the adS/CFT correspondence, for which an extension to non-conformal cases was considered in [15]. One purpose of this paper is to show how the dual frame allows a uniform discussion of both the conformal and non-conformal cases. The conformal cases are those for which the dual frame is self-dual. In all cases an essential ingredient of the correspondence between the supergravity theory and the QFT is that the conserved currents of the QFT couple to the bulk supergravity fields. One therefore needs to specify some embedding of the brane worldvolume in the bulk spacetime. In other words, where do we put the branes?

In the adS/CFT correspondence, it is natural to place the CFT at the adS boundary because this hypersurface is a fixed surface under the action of the conformal group on the adS spacetime. The only other hypersurface with this property is the horizon. However, one can define a Minkowski space QFT with non-linearly realized conformal invariance on any horosphere [16]. The adS_{p+2} metric in horospherical

[3] As against the 'external' dynamics of the brane's motion in the D=10 or D=11 spacetime, which is described by the Born-Infeld type brane actions for the Goldstone modes of broken supertranslations.

coordinates is

$$ds^2 = u^2 ds^2\left(\mathbb{E}^{(p,1)}\right) + u^{-2} du^2\,, \tag{1.1}$$

and the horospheres are the hypersurfaces of constant u. A typical horosphere is shown on the Carter-Penrose diagram of adS_{p+2} in the figure below.

The horospheres form a one-parameter class of Minkowski hypersurfaces interpolating between the Killing horizon and the adS boundary, the parameter being the radial adS coordinate. A scale transformation shifts the value of this radius, i.e. it takes a QFT in one vacuum to a QFT in another vacuum. In the IR or UV limit one is driven to the adS horizon or its boundary, respectively, these being the two fixed points in the space of vacua. Almost the same picture carries over to the domain-wall/QFT correspondence with the difference that the choice of horosphere for the Minkowski vacuum now corresponds not to a choice of the vacuum of a QFT with non-linearly realized conformal symmetry but rather to a choice of coupling constant of a non-conformal QFT. Thus, in the non-conformal case the interpolation between the adS Killing horizon and its boundary corresponds to an interpolation between strong and weak coupling (or vice-versa, depending on the case). Since the coupling constant is dimensionful this is again equivalent to a choice of scale.

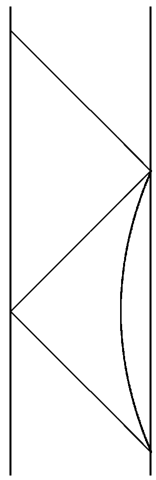

Figure 1: Carter-Penrose diagram of anti-de Sitter spacetime. The diagonal lines are Killing horizons. The curved line is a horosphere.

In the adS/CFT correspondence, operators of the CFT couple to fields of the KK supergravity about the adS background, and correlation functions of these operators are related to classical solutions of the KK supergravity equations with boundary conditions specified at the adS boundary. We expect the same to be true in the domain-wall/QFT correspondence but the graviton in this background no longer belongs to the graviton supermultiplet of an adS-supergravity. Rather, it must belong to the graviton supermultiplet of a class of gauged supergravity theories for which the vacuum is instead a 1/2 supersymmetric domain wall spacetime. We further expect the QFT on the worldvolume of coincident Dp-branes to be encoded in a KK supergravity theory with a 1/2 supersymmetric domain wall vacuum and another purpose of this paper is to identify the relevant lower-dimensional gauged supergravity theory in each case. We shall see that, in each case, the gauge group of the supergravity theory coincides with the R-symmetry group of the equivalent QFT.

2. 'Near-horizon' limit in the dual frame

We shall be interested in a limit of (IIA, IIB, or N=1) superstring theory for which the dynamics is well-approximated by an effective supergravity action. The part of this action relevant to the Dp-brane contains the metric, dilaton and a Ramond-Ramond field strength, which can be either a $(p+2)$-form (in which case the Dp-brane solution is 'electric') or a $(8-p)$-form (in which case the Dp-brane solution is 'magnetic'). For the latter choice, and in the string frame, the action is

$$S = \frac{1}{\alpha'^4} \int d^{10}x \sqrt{-g} \left[e^{-2\phi} \left(R + 4(\partial\phi)^2 \right) - \frac{1}{2(8-p)!} |F_{8-p}|^2 \right] . \tag{2.1}$$

The validity of the supergravity approximation requires that we take the limits

$$\alpha' \to 0 \qquad g_s \to 0 \,, \tag{2.2}$$

where α' is the inverse string tension and $g_s = e^{\phi_\infty}$ is the string coupling constant (with ϕ_∞ the asymptotic value of the dilaton field ϕ).

The equations of motion of the above action have the solution

$$\begin{aligned} ds^2_{st} &= H^{-1/2} ds^2(\mathbb{E}^{(p,1)}) + H^{1/2} ds^2(\mathbb{E}^{(9-p)}) \,, \\ e^{\phi} &= g_s H^{(3-p)/4} \,, \\ F_{8-p} &= g_s^{-1} \star dH \,, \end{aligned} \tag{2.3}$$

where $\star$ is the Hodge dual of $\mathbb{E}^{(9-p)}$ and H is harmonic on $\mathbb{E}^{(9-p)}$. Let r be the distance from the origin of $\mathbb{E}^{(9-p)}$. The choice

$$H = 1 + g_s N \left(\frac{\sqrt{\alpha'}}{r} \right)^{(7-p)} \,, \tag{2.4}$$

then yields the long-range fields of N infinite parallel planar Dp-branes near the origin.

We now consider the low energy limit[1, 15], $\alpha' \to 0$, keeping fixed the mass of stretched strings, $U = r/\alpha'$ (so $r \to 0$), and all (other than r) coordinates that appear in (2.3). In addition, we hold fixed the 't Hooft coupling constant,

$$g^2_{YM} N = \text{fixed} \,, \tag{2.5}$$

where

$$g^2_{YM} = g_s (\alpha')^{(p-3)/2} \,. \tag{2.6}$$

This means that we must have $g_s N \to 0$ for $p < 3$ and $g_s N \to \infty$ for $p > 3$. For $p < 3$ this requirement is satisfied for any N because we take $g_s \to 0$. For $p > 3$ it can be satisfied only if we also take the limit $N \to \infty$. For N finite, (2.5) is equivalent to keeping energies finite on the worldvolume while taking the low-energy limit. The $N \to \infty$ limit is the 't Hooft limit[17]. In either case, we now have

$$H = 1 + \frac{g^2_{YM} N}{(\alpha')^2 U^{(7-p)}} \to g^2_{YM} N (\alpha')^{-2} U^{(p-7)} \,. \tag{2.7}$$

The string metric of (2.3) is singular at $U=0$, so it is not yet clear whether the above 'near-horizon' limit yields a limiting supergravity solution in the way that it does in the D3-brane case [6]. We can circumvent this problem by considering the 'dual frame' metric

$$g_{dual} = (e^{\phi}N)^{\frac{2}{p-7}} g_{st} \,. \tag{2.8}$$

In this frame the action (2.1) is

$$S = \frac{N^2}{\alpha'^4}\int d^{10}x\,\sqrt{-g}(Ne^{\phi})^{\gamma}\left[R + \frac{4(p-1)(p-4)}{(7-p)^2}(\partial\phi)^2 - \frac{1}{2(8-p)!}\frac{1}{N^2}\left|F_{8-p}\right|^2\right], \tag{2.9}$$

where

$$\gamma = 2(p-3)/(7-p)\,. \tag{2.10}$$

Note that the dual field strength couples to the dilaton in the same way as the metric; hence the terminology 'dual frame'. For $p=3$ the dual frame coincides with the Einstein frame up to a power of N. The Dp-brane dual frame metric is

$$ds^2_{dual} = (g_s N)^{2/(p-7)}\left[H^{(5-p)/(p-7)}ds^2(\mathbb{E}^{(p,1)}) + H^{2/(7-p)}ds^2(\mathbb{E}^{(9-p)})\right], \tag{2.11}$$

for which the singularity at $U=0$ is now just a coordinate singularity. The full 'near-horizon' solution is

$$\begin{aligned}
ds^2_{dual} &= \alpha'\left[\left(g^2_{YM}N\right)^{-1}U^{5-p}ds^2\left(\mathbb{E}^{(p,1)}\right) + U^{-2}dU^2 + d\Omega^2_{(8-p)}\right],\\
e^{\phi} &= \frac{1}{N}\left[\left(g^2_{YM}N\right)U^{p-3}\right]^{(7-p)/4},\\
F_{8-p} &= (7-p)N(\alpha')^{(7-p)/2}vol\left(S^{8-p}\right).
\end{aligned} \tag{2.12}$$

The near-horizon metric is $adS_{p+2}\times S^{8-p}$ when $p\neq 5$. When $p=5$ it is $\mathbb{E}^{(6,1)}\times S^3$, so we exclude this case in what now follows. We will return to the $p=5$ case later.

It is important to notice that all factors of α' cancel out at the end: in the string worldsheet action the overall α' in the metric cancels against the α' in the string tension, and in the effective supergravity action the factors of α' coming from the factors of α' in (2.12) cancel against the α' in Newton's constant. This cancellation allows us to now set $\alpha'=1$ in what follows.

The adS metric can be put in a standard form by the introduction of a new radial coordinate u defined by

$$u^2 = \mathcal{R}^2\left(g^2_{YM}N\right)^{-1}U^{5-p}\,,\qquad [\mathcal{R} = 2/(5-p)]\,. \tag{2.13}$$

Since both U and $g^2_{YM}N$ remain finite in the near-horizon limit, so too does u. We now have

$$\begin{aligned}
ds^2_{dual} &= \frac{u^2}{\mathcal{R}^2}ds^2\left(\mathbb{E}^{(p,1)}\right) + \mathcal{R}^2\frac{du^2}{u^2} + d\Omega^2_{(8-p)}\,,\\
e^{\phi} &= \frac{1}{N}\left(g^2_{YM}N\right)^{(7-p)/2(5-p)}(u/\mathcal{R})^{(p-7)(p-3)/2(p-5)}\,,\\
F_{8-p} &= (7-p)N\,vol\left(S^{8-p}\right).
\end{aligned} \tag{2.14}$$

We see from this form of the metric that $\mathcal{R}$ is the adS radius of curvature. The hypersurface $u = 0$ is a non-singular Killing horizon while the boundary of the adS space is at $u = \infty$.

There is an UV/IR connection between the bulk and the boundary theory. Specifically, it was shown in [18] that the holographic energy scale of the boundary QFT is

$$E \sim \frac{U^{(5-p)/2}}{g_{YM} N^{1/2}} \,. \tag{2.15}$$

This energy-distance relation leads to a holographic result for the number of states of string theory in *adS* space [19, 18]. From (2.13) we see that it is equivalent to

$$E \sim u \,. \tag{2.16}$$

In other words, the scale u introduced by the requirement that the *adS* metric in the dual frame take the same form for all p (including the conformal $p = 3$ case) is the holographic energy scale of the boundary QFT! Thus, one may argue that the dual-frame is the "holographic" frame describing supergravity probes (the string frame being associated with D-brane probes).

The gauge theory description is valid provided that the effective dimensionless YM coupling constant (at the energy scale of interest)

$$g_{eff} = g_{YM} N^{1/2} E^{(p-3)/2} \,, \tag{2.17}$$

is small. Using (2.16)-(2.15) we see that this requires

$$\left[g_{YM}^2 N U^{p-3}\right]^{(5-p)} \ll 1 \,. \tag{2.18}$$

On the other hand, string perturbation theory is valid when the dilaton (2.14) is small. This is always true in the large N limit, except near $u = 0$ for $p < 3$ and $p = 6$ or $u = \infty$ for $p = 4$. Depending on the rate with which we approach these points SYM or supergravity will be valid in some region of the parameter space, but eventually the string coupling grows large and we have to pass to the strong string-coupling dual.

The validity of the supergravity solution requires that the effective string tension times the characteristic spacetime length is large. The latter can be read-off from (2.14) and is of order 1. Therefore, we get the condition

$$T_{dual} = (N e^{\phi})^{2/(7-p)} \gg 1 \,, \tag{2.19}$$

which implies

$$g_{YM}^2 N U^{p-3} \gg 1 \,, \tag{2.20}$$

for the validity of the supergravity description of the D-brane dynamics. This is the same as the condition found in [15, 18] by requiring the curvatures in the string frame to remain small.

The above conditions and their implications have been discussed in detail in [15, 18]. We provide a brief summary here for $p = 0, 1, 2, 4, 6$, as this will be helpful for the discussion to follow of the associated supergravity domain wall spacetimes. The $p = 0, 1, 2$ cases are similar in that the region close to the boundary (corresponding to the UV of the SYM) is described in terms of perturbative SYM since the effective coupling constant is small there. The IR limit of the SYM theory, which corresponds (in the dual frame) to the region near the horizon at $u = 0$, is a strong coupling limit because g_{eff} is large there. The effective string coupling constant e^{ϕ} is also large in this region, so the horizon limit is the strong string coupling limit. This means that to resolve the singularity in e^{ϕ} at the horizon one must pass to the strong coupling dual theory. For $p = 0, 2$ this is M-theory while for $p = 1$ it is the dual IIB theory. In the latter case, after S-duality the D-string supergravity solution becomes the fundamental string solution. The dilaton is then small near $u = 0$ but there is now a curvature singularity there which is resolved in the DVV matrix string theory [20]. In the $p = 4$ case the region close to the boundary again corresponds to the UV of the SYM theory but this theory is now strongly coupled there. The $D = 10$ supergravity description is valid for sufficiently large N at any given distance from the boundary but for any N, no matter how large, the string coupling blows up as the adS boundary is approached. The $D = 10$ supergravity description therefore fails in the latter limit but a description in terms of $D = 11$ supergravity may be valid because the singularity of the dilaton at the boundary is resolved by the interpretation of adS_6 with a 'linear' dilaton as adS_7 [14]. Near the adS horizon the supergravity description fails but the SYM description is valid because the effective gauge coupling is small.

The $p = 6$ case is rather different because the supergravity description is apparently valid in the same near-horizon region in which the SYM theory is weakly coupled. This would seem to lead to a contradiction, but the issue of decoupling gravity in the case of $D6$ branes is rather intricate [21, 22].

3. Gauged supergravities and domain-walls

The dual frame formulation is natural from the supergravity point of view because the factorization of the geometry leads to an immediate identification of the lower dimensional gauged supergravity that the graviton is part of.

From the solution (2.14) we see that there is an S^{8-p} compactification of the D=10 theory to an effective gauged $(p + 2)$-dimensional supergravity with action of the form

$$S = N^2 \int d^{p+2}x \sqrt{-g} \left(Ne^{\phi}\right)^{\gamma} \left[R + \frac{4(p-1)(p-4)}{(7-p)^2}(\partial\phi)^2 + \frac{1}{2}(9-p)(7-p)\right] . \tag{3.1}$$

where γ is the constant given in (2.10). We also deduce that the field equations of this action admit an adS_{p+2} vacuum with 'linear' dilaton (as explained in [14], the dilaton can be invariantly characterized in terms of a conformal Killing potential).

On passing to the Einstein frame (for $p \neq 0$) we find the action

$$S = N^2 \int d^{p+2}x \sqrt{-g} \left[R - \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(9-p)(7-p)N^b e^{a\phi} \right], \tag{3.2}$$

where

$$a = \frac{-\sqrt{2}(p-3)}{\sqrt{p(9-p)}}, \qquad b = \frac{4(p-3)}{p(p-7)}. \tag{3.3}$$

In this frame the adS_{p+2} linear dilaton vacuum is equivalent to the domain wall solution studied in [23, 24, 8]. The parameter Δ used in [8] to characterize various kinds of domain wall solutions is in our case ($D = p+2$),

$$\Delta \equiv a^2 - \frac{2(D-1)}{D-2} = \frac{-4(7-p)}{9-p}, \tag{3.4}$$

with $\Delta_{\rm adS} = \frac{-2(D-1)}{D-2}$ the value corresponding to the effective Lagrangian which admits anti-de Sitter spacetime as a solution. As expected, $\Delta = \Delta_{\rm adS}$ only for $p = 3$. For all other p the domain wall is in the category $\Delta_{\rm adS} < \Delta < 0$ [8]. We now turn to an application of the above analysis to Type II D-p-branes. The values of p to which the above analysis is applicable are $p = 0, 1, 2, 4, 6$. We exclude $p = 3$ as the domain wall vacuum is then the well-known supersymmetric adS vacuum. We shall consider the $p = 5$ case later.

3.1 p-branes, $p \neq 5$

From the $p = 0$ near-horizon limit we see that there is an S^8 compactification of IIA supergravity to a D=2 $SO(9)$ gauged maximal supergravity theory. The R-symmetry group of the corresponding D=1 QFT is expected to be the largest subgroup of $SO(16)$ with a 16-dimensional spinor representation [25]; this is precisely $SO(9)$. Since very little is known about gauged supergravity theories in D=2 we turn now to $p = 1$. For $p = 1$ we see that there is an S^7 compactification of, for example, IIA supergravity to a D=3 $SO(8)$ gauged maximal supergravity. This is obviously the S^1 reduction of the de Wit-Nicolai $SO(8)$ gauged N=8 supergravity in D=4; as shown in [8], the adS_4 vacuum of the latter will descend to a domain wall solution in D=3. In the IIB case the interpretation of the $SO(8)$-gauged D=3 supergravity as a reduction of the de Wit-Nicolai theory is not available, and it may well be a different theory; gauged D=3 supergravities have not yet been systematically explored. Furthermore, from the type I fundamental string solution we expect that there should be a truncation of maximal $SO(8)$-gauged D=3 supergravity to a half-maximal one with the same gauge group. In all of these cases the corresponding D=2 QFT has the expected $SO(8)$ R-symmetry group (identifying the left and right $SO(8)$ groups).

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We now turn to $p = 2$. In this case we find an S^6 compactification of IIA supergravity which we can also view as an $S^6 \times S^1$ compactification of D=11 supergravity.[4] The effective gauged D=4 supergravity must be one of those found by Hull as these have recently been shown to be complete [27]. Since the isometry group of S^6 is $SO(7)$ we might expect the gauge group to be $SO(7)$ (which is also the R-symmetry group of the D2-brane). The obvious candidate is the $ISO(7)$ gauged supergravity [28] because only the $SO(7)$ subgroup can be linearly realized. Moreover, this theory has a potential [29] of the right form to admit a 1/2 supersymmetric domain wall solution. The correctness of this identification follows from the observations [30] that the non-compact gauged N=8 supergravity theories can be obtained by 'compactification' of D=11 supergravity on hyperboloids of constant negative curvature, and that the contracted versions, such as $ISO(7)$, correspond to a limit in which the hyperboloid degenerates to an infinite cylinder. The $ISO(7)$ theory thus corresponds to a 'compactification' on the cylinder $S^6 \times \mathbb{E}^1$, but we may replace this by $S^6 \times S^1$. The reason that the near-horizon limit of the D2-brane differs from that of the M2-brane (for which the effective D=4 theory is the $SO(8)$ gauged de Wit-Nicolai theory) is that the M2-brane harmonic function is harmonic on $\mathbb{E}^8$ whereas the D2-brane harmonic function is harmonic on $\mathbb{E}^7$. Further dimensional reduction will lead to functions that are harmonic on $\mathbb{E}^{8-k}$ and hence to new 'near-horizon' limits, for which the effective D=4 theory is a $CSO(8-k,k)$ gauged supergravity (in the notation of [31]).[5]

Passing over $p = 3$ we come to $p = 4$. In this case we find an S^4 compactification of IIA supergravity. As mentioned earlier, this is just the S^4 compactification of D=11 supergravity in disguise. This is consistent with the fact that the $SO(5)$ gauged maximal D=6 supergravity is just the reduction on S^1 of the $SO(5)$ gauged maximal D=7 supergravity [32]. It is also consistent with the fact that the R-symmetry group of the D4-brane is $SO(5)$.

Passing over $p = 5$ we come to $p = 6$. Despite the problematic features of the correspondence in this case, consideration of the 'near-horizon' limit of the D6-brane dual-frame solution still allows us to deduce the existence of an S^2 compactification of IIA supergravity to an $SU(2)$ gauged D=8 supergravity (note that $SU(2)$ is also the R-symmetry group of the D6-brane). We shall now argue that this theory is the one found by Salam and Sezgin [12] from a generalized Scherk-Schwarz reduction on an SU(2) group manifold of D=11 supergravity.

In the S^2 compactification of IIA supergravity there is a 2-form field strength proportional to the volume form on S^2. But this 2-form is also the field strength of the KK gauge field arising in the S^1 compactification of D=11 supergravity. It follows

[4] This is not included in a previous classification [26] of compactifying solutions of D=11 supergravity to D=4 because the D=4 spacetime is a domain wall spacetime rather than adS.

[5] The linearly realized gauge group in this case is $SO(8-k)$; one would therefore expect the (non-conformal) interactions of the associated D=3 QFT to break $SO(8)$ to $SO(8-k)$.

that the S^2 compactification of IIA supergravity is equivalent to a compactification of D=11 supergravity on a $U(1)$ bundle over S^2. For unit charge this is just the Hopf fibration of S^3. As confirmation of the equivalence of the two compactifications we now observe that the gravity/dilaton sector of the effective gauged D=8 supergravity arising from an S^2 compactification of IIA supergravity is, according to (3.2),

$$S = \int d^8x \sqrt{-g} \left[R - \frac{1}{2}(\partial\phi)^2 + \frac{3}{2}N^{-2}e^{-\phi} \right] . \tag{3.5}$$

This has exactly the same dilaton potential as the Salam-Sezgin action if all the other scalar fields appearing in the latter are set to zero, and if one identifies the SU(2) coupling constant g with $2N^{-2}$.

The domain wall solution of the field equations of (3.5) is

$$\begin{aligned} ds^2 &= e^{\frac{1}{2}\rho} ds^2\left(\mathbb{E}^{(6,1)}\right) + N^{-2} e^{\frac{3}{2}\rho} d\rho^2 \,, \\ \phi &= \frac{3}{2}\rho \,, \end{aligned} \tag{3.6}$$

where we use a coordinate ρ for which the dilaton is linear. This is the domain wall solution of [8] for $D = 8$ and $\Delta = -\frac{4}{3}$. It is easily shown to be a 1/2 supersymmetric solution of the $SU(2)$ gauged D=8 supergravity by using the supersymmetry transformation rules given in [12]. On the other hand, the same solution can be found from the near-horizon limit (2.12) of the D6 brane solution by splitting off the (compact) internal part and passing to the Einstein frame.

3.2 Fivebranes

Returning to (2.12) for $p = 5$, defining

$$\rho = \log\left(g_{YM} N^{1/2}\, U\right) \,, \tag{3.7}$$

and rescaling the worldvolume coordinates by $g_{YM}N^{1/2}$ we have the 'near-horizon' solution

$$\begin{aligned} ds^2_{dual} &= \alpha' \left[ds^2\left(\mathbb{E}^{(5,1)}\right) + d\rho^2 + d\Omega_3^2 \right] \\ \phi &= \rho - \log N \\ F_3 &= 2N\alpha'\, vol(S^3) \end{aligned} \tag{3.8}$$

This is $\mathbb{E}^{(6,1)} \times S^3$ with a linear dilaton. Minkowski space does not have a boundary, so the issue of holography for fivebranes is more intricate [15, 33, 18]. Nevertheless, we deduce, as in the previous cases, that there is an S^3 compactification to D=7 [6]. In the context of N=1 D=10 supergravity the resulting D=7 supergravity theory has been identified as the $SU(2)$ gauged theory of [11], coupled to an $SU(2)$ SYM theory, with the D=7 vacuum being the domain wall solution found in [24].

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The same S^3 compactification is a solution of IIA and IIB supergravity, now arising as the 'near-horizon' limit of the IIA or IIB NS-5-brane. Let us first consider the IIA case, which can be viewed as an $S^3 \times S^1$ compactification of D=11 supergravity. From our analysis of the $S^6 \times S^1$ compactification in the previous section we would expect the effective D=7 theory to be an $ISO(4)$ gauged D=7 maximal supergravity. No such theory is currently known but there is an $SO(4,1)$ gauged D=7 maximal supergravity [34]. There is also an $SO(3,2)$ theory. The known list cannot be complete, however, because it does not include the $SU(2)$ gauged theory found by S^1 reduction of the $SU(2)$ gauged D=8 theory; the former is probably a contraction of the $SO(3,2)$ D=7 theory. It therefore seems possible that there is an $ISO(4)$ theory awaiting construction, and that this theory is the effective D=7 theory for the KK compactification of IIA supergravity provided by the near-horizon limit of the NS-5-brane.[6]

In the IIB case the effective D=7 theory may well be a different (as yet unknown) $SO(4)$-gauged D=7 maximal supergravity theory. Note that the $SO(4)$ gauge group is expected not only from the fact that this is the isometry group of S^3 but also from the fact that the R-symmetry group of the IIB NS-5-brane is $SO(4)$. Of course the same analysis applies to the D5-brane, in which case it is clear that the QFT in the domain-wall/QFT correspondence should be a D=6 $SU(N)$ SYM theory on a hypersurface of constant ϕ in the domain-wall spacetime. As ρ varies from $-\infty$ to ∞ the effective coupling constant varies from zero (weak coupling) to infinity (strong coupling). In contrast to the $p \neq 5$ cases discussed previously, there is now *no asymmetry between the weak and strong coupling limits*. Unlike the $p \neq 5$ cases, there is now an $\rho \to -\rho$ isometry of the near-horizon spacetime.

As a final observation concerning fivebranes we note that there is a generalization of the S^3 compactification of D=10 supergravity theories to an $S^3 \times S^3$ compactification which can be interpreted [9] as the near-horizon limit of intersecting 5-branes. In the context of N=1 supergravity (or the heterotic string theory) the $S^3 \times S^3$ compactification yields the $SO(4)$ gauged Freedman-Schwarz model [35, 36]. The same solution can be used to compactify either IIA or IIB supergravity, and one may then wonder what the effective D=4 theory is in these cases. The only obvious candidate is the $SO(4,4)$ gauged D=4 maximal supergravity of [31]. Unlike the $SO(8)$ theory, it can be truncated to the FS model. The $SO(4,4)$ theory has a (non-supersymmetric) de Sitter vacuum that has been interpreted as a 'compactification' of D=11 supergravity on a hyperboloid [30], but this does not preclude the possibility of other solutions of the same theory having quite different higher-dimensional interpretations.

[6] On the other hand, the R-symmetry group of the IIA 5-brane is $SO(5)$, which is larger than the linearly realized $SO(4)$ subgroup of $ISO(4)$.

4. Comments

We have proposed that superstring or M-theory in certain domain wall spacetimes is equivalent to a quantum field theory describing the internal dynamics on N coincident branes. The near-horizon limit of the corresponding brane supergravity solution yields a compactification to the domain wall spacetime of the proposed equivalence. This proposal extends the usual adS/CFT correspondence by viewing the adS spacetime as a special case of a domain wall spacetime.

In the course of formulating this proposal we have added to the list of gauged supergravity theories that can be interpreted as 'near-horizon' limits of brane, or intersecting brane solutions of M-theory or superstring theories (in this paper we considered single brane solutions, but we expect that a similar analysis can be performed for intersecting branes as well). Notably, we have found a role for the $SU(2)$-gauged maximal D=8 supergravity, and the $ISO(7)$ gauged N=8 theory in D=4. As observed earlier, the last case is just one in the $CSO(8-k,k)$ series of gauged N=8 D=4 supergravity theories that can be found as near-horizon limits of the T^k reduction of the M2-brane. Toroidal reductions of other branes can be expected to lead to similar series of non-compact gaugings in other dimensions. For example, T^k reductions of the M5-brane and the IIB 3-brane are expected to be related to new $CSO(5-k,k)$ gauged D=7 supergravity theories and $CSO(6-k,k)$ gauged D=5 supergravities respectively (the known non-compact gaugings of maximal D=5 supergravity can be found in [37]). Several of these supergravities are expected to be related to each other by dimensional reduction since the corresponding brane configurations are related by T-dualities. As yet there is no known brane interpretation of the non-contracted versions of these non-compact gauge groups.

One intriguing aspect of our results is the association of the dual frame metric with the holographic energy scale in the boundary QFT. The fact that the dual brane has dilaton independent tension in this frame indicates that elementary $(6-p)$ branes may play a role. This possibility was ruled out for toroidal compactifications in [38] but spherical compactifications might lead to a different conclusion.

A prerequisite of any domain-wall/QFT correspondence is that the R-symmetry of the supersymmetric QFT on the domain wall worldvolume match the gauge group of the equivalent gauged supergravity. In this paper we have seen evidence this requirement is met by all p-branes with $p \leq 6$, the restriction on p arising from the fact that for $p > 7$ the harmonic functions are not bounded at infinity. Nevertheless, the fact that branes with $p > 7$ are connected via dualities to $p < 7$ branes suggests that a similar story should hold for $p \geq 7$. For example, the worldvolume field theory of a 7-brane has an $SO(2)$ R-symmetry, which suggests the existence of an $SO(2)$-gauged $D = 9$ maximal supergravity admitting a 1/2 supersymmetric domain wall vacuum. The D=9 duality group is $Gl(2;\mathbb{R})$, which is just big enough to allow for an $SO(2)$ gauging. The construction of this model will be described in a future publication [39]. The remaining two cases, $p = 8$ and $p = 9$, also fit the pattern. In the $p = 8$ case the domain wall is the D-8-brane of the massive IIA supergravity. The R-symmetry group of the field theory on the domain wall is trivial as is the gauge group of the bulk IIA supergravity. In the $p = 9$ case the domain wall is the 'M-boundary' of the Hořava-Witten theory [40]. The R-symmetry group is again trivial, as is the gauge group of the bulk D=11 supergravity theory.

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